On Models for Transdisciplinarity

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doi: 10.22545/2011/00017

In this paper we discuss about transdisciplinarity, interdisciplinarity, pluridisciplinarity, and unification theories in mathematics. Some related duality extensions are also presented.

Keywords*:* transdisciplinarity, coalgebra theory, algebraic model.

1 Introduction

The transdisciplinarity ([1,2]) is a new approach about disciplines and what is between disciplines, above them and beyond them. Its purpose is the understanding of the current world. For example, this paper, written at a transdisciplinary level, abides somewhere between epistemology and abstract algebra, with implications in physics, topology, etc.

In abstract algebra, two diferent algebraic structures, which are dual concept, were unified at another level by [3]. This was posible by embedding them in the category of the Yang-Baxter structures. (Recasting some objects in another setting, in order solve certain problems is a nonstandard tehnique in mathematics.) The celebrated Yang-Baxter equation traverses statistical mechanics, theoretical physics ([4]), knot theory ([5]), quantum qroups ([6]), etc.

Similarly, one can view the transdisciplinarity as a unification for interdisciplinarity and pluridisciplinarity. Our analogy is based on the observation that interdisciplinarity appears at the border of two different disciplines, while in pluridisciplinarity we deal with several disciplines serving a certain discipline.

Consequently, the current paper attemps to clarify the transdisciplinary terminology for the interested mathematicians, gives an informal introduction to the coalgebra theory and proposes the use of mathematical models in the development of the transdisciplinary thinking. The organization of the paper is the following. In section 2 we detail our algebraic model for transdisciplinarity. The third section contains algebraic details about duality extensions and explanations.

2 An algebraic model

Let us consider Figure 1, which shows that the transdisciplinarity includes both the interdisciplinarity and pluridisciplinarity. Let us detail this picture. The interdisciplinarity generates new disciplines. For example, the transfer of the mathematical models in physics generated the mathematical physics. (For examples of interactions between mathematics and music, or between mathematics and lingvistics, we refer to [7].)

Let us use the mathematical formalism to describe this situation. For the disciplines

$$D_{-1} \text{ and } D_{-2} \tag{1}$$

The interdisciplinarity associates a (new) discipline:

$$D = D_{-1} \bullet D_{-2} \tag{2}$$

In mathematics,

$$D \bullet 1 = 1 \bullet D = D \qquad \forall D \tag{3}$$

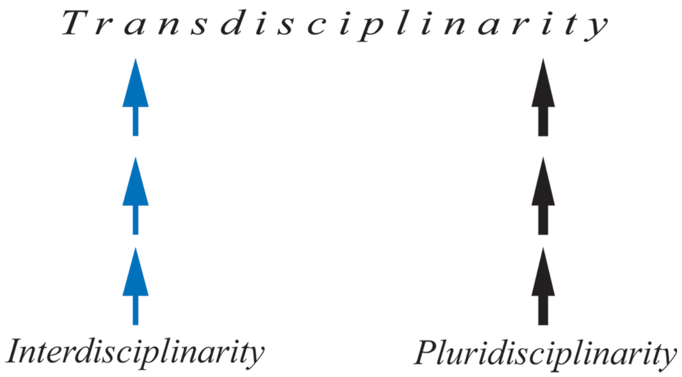

Figure 1: Interdisciplinarity and Pluridisciplinarity.

In our case, a unity is represented by an "empty disciple", a discipline which only contains the notions, symbols and formulas appearing in all disciplines.

The pluridisciplinarity refers to the study of an object from one discipline, using other disciplines. For example, a Giotto's picture can be studied from the perspective of art history, physics, chemistry, history of religions, history of Europe and geometry (cf. [1]).

Let us use the mathematical formalism to describe this situation. We consider one object from a discipline

$$0 \in D \tag{4}$$

and take projections of it into other disciplines

$$0_i \in D \qquad \forall_i \in I. \tag{5}$$

In mathematics, this is called a co-operation or comultiplication.

Now, the information processed by these new disciplines will be collected and help to evaluate the initial object. In mathematics, this role is played by the co-unity. Back to Abstract Algebra, we consider the category of rings or, even better, algebras to represent the interdisciplinarity. (An algebra structure has a multiplication and a unity.) The category of coalgebras, or co-rings, could be used as a model for pluridisciplinarity. (A coalgebra has a comultiplication and a counity.)

Now, the question is if the category of algebra structures and the category of coalgebra structures can be seen as forming just one "bigger" category. This later category would play the role of the transdisciplinarity.

A unification for the categories of algebras and coalgebras was proposed in [8]. Moreover, in a special case, this unification is also an extension for the duality between algebras and coalgebras. This "bigger" category is related to the celebrated Yang-Baxter equation. Figure 2 briefly explains this unification.

The final conclusions of this section are related to the interactions between mathematics and physics, which are not only described by the mathematical-physics, but are of a more complex nature: many problemes arising in physics helped the development of mathematics; on the other hand, by solving equations from physics, the mathematicians help the physicists, and sometimes anticipate their observations.

This kind of situations are suggested by the two diagrams of this section.

3 Algebraic Details

This section is devoted to Abstact Algebra, and its purposes is to briefly explain to the reader the concepts used before and to give him a short bibliography.

For the algebraic aspects of the Yang-Baxter equation, we recomand the book [9]. Algebras, coalgebras and their duality are studied in [10]. A more compact lecture on these topics could be [8] or, alternatively, [3] and [11].

The Pontryagin duality refers to the duality between the categories of compact Hausdorff Abelian groups and discrete Abelian groups. The Pontryagin-van Kampen duality theorem extends this duality to all locally compact Hausdorff Abelian groups (see [12]) and it also represents a unification for the two kinds of topological groups.

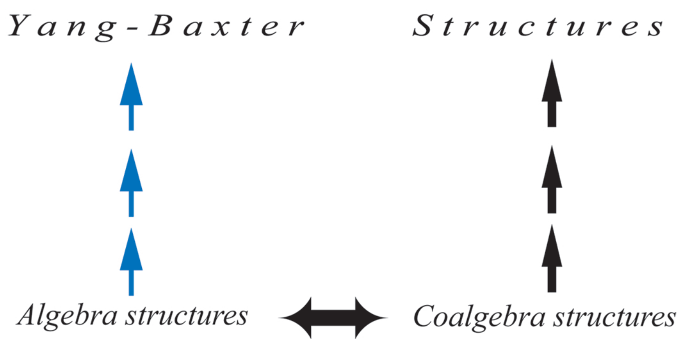

Figure 2: Unification for the Categories of Algebras and Coalgebras.

Taking the Pontryagin-van Kampen duality theorem as a model, we posed the following question: ≪ Is it possible to extend the duality between finite dimensional algebras and coalgebras in the same spirit?≫ We gave a positive answer by constructing the extension described in the previous section.

The constructions related to the extension of the duality between finite dimensional algebras and coalgebras have many applications: in noncommutative descent theory ([13]), in constructing large classes of Yang-Baxter operators([14]) and Yang-Baxter systems ([15]), in Knot Theory ([5]), in finding solutions for the colored Yang-Baxter equation ([16]), etc.

4 Conclusions

Algebras and coalgebras are different algebraic structures, and their axioms are dual to each other. While the algebras have multiplications, the coalgebras have another type of operations, called comultiplications. Therefore, the unification of these structures came as a surprise. This unification was possible by recasting these structures in the category of solutions for the Yang-Baxter equation. Thus, this equation captures the common information encapsulated in the associativity axiom and in the coassociativity axiom.

The interdisciplinarity associates a discipline for other two disciplines, and the pluridisciplinarity refers to the study of an object from one discipline, using other disciplines. The transdisciplinarity is a more general approach about disciplines. It includes the interdisciplinarity and pluridisciplinarity; the transdisciplinary thinking motivates the attempts of unifying theories, structures, disciplines, etc.

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ISSN: 1949-0569 online